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6. AUTHOR(S)
Giulio Casati

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Centro di Cultura Scientifica A Volta
Villa Olmo- Via Cantoni, 1
22100 COMO Italy

8. PERFORMING ORGANIZATION REPORT NUMBER GC1-2002

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
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13. ABSTRACT (Maximum 200 words)
We have shown the possibility to construct a CNOT gate for free fermions, using only beam splitters and spin rotations . We have predicted that the annihilation of an electron and a hole by elastic scattering leads to teleportation of the (unknown) state of the annihilated electron to a second, distant electron -- if the latter was previously entangled with the annihilated hole.
We have constructed efficient quantum algorithms for the simulation of complex quantum systems and have studied their stability under noisy perturbations and systems imperfections. Finally we have clarified the relation between chaos and entanglement.

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Statement of the problem

Quantum computation is an emerging interdisciplinary field that takes advantage of concepts from both information theory and quantum mechanics. Indeed, a quantum computer can be seen as a complex many-body interacting system. In the last two decades, a great effort has been made and considerable achievements have been obtained in the understanding of the properties of complex quantum systems. In particular, the quantum mechanical manifestations of the onset of chaos in classical mechanics have been analyzed. These results found applications in different physical fields, from atomic physics to mesoscopic physics and quantum optics. From a theoretical viewpoint, this favoured the development of various numerical and analytical techniques. Typical problems in the field of quantum chaos are decoherence and the quantum to classical transition, subjects that are also essential for any realistic implementation of a quantum computer. The purpose of the present research is to use this knowledge and the various methods developed in the study of quantum systems to face important problems in quantum computation.

Summary of the most important results

<u>Free-electron quantum computation</u>: It is known that a quantum computer operating on electron-spin qubits with single-electron Hamiltonians and assisted by single-spin measurements can be simulated efficiently on a classical computer. We have shown that the exponential speed-up of quantum algorithms is restored if single-charge measurements are added. These enable the construction of a CNOT gate for free fermions, using only beam splitters and spin rotations. The gate is nearly deterministic if the charge detector counts the number of electrons in a mode, and fully deterministic if it only measures the parity of that number.

<u>Electron-hole pair entanglement</u>: We have demonstrated theoretically that the shot noise produced by a tunnel barrier in a two-channel conductor violates a Bell inequality. The non-locality is shown to originate from entangled electron-hole pairs created by tunneling events -- without requiring electron-electron interactions. A pair of edge channels in the quantum Hall effect is proposed as experimental realization.

<u>Spin-entangled electron-hole pair pump</u>: A non perturbative theory is presented for the creation by an oscillating potential of spin-entangled electron-hole pairs in the Fermi sea. In the weak potential limit, the entanglement production is much less than one bit per cycle. We demonstrate that a strong potential oscillation can produce an average of one Bell pair per two cycles, making it an efficient source of entangled flying qubits.

<u>Quantum teleportation by particle-hole annihilation in the Fermi sea:</u> We have predicted that the annihilation of an electron and a hole by elastic scattering leads to teleportation of the (unknown) state of the annihilated electron to a second, distant electron -- if the latter was previously entangled with the annihilated hole. We have proposed an experiment, involving low-frequency noise measurements on a two-dimensional electron gas in a high magnetic field, to detect teleportation of electrons and holes in the two lowest Landau levels.

Quantum algorithms: We have studied the quantum dynamical localization for a dynamical model- the sawtooth map- using an algorithm that we have previously developed. This algorithm has some specific advantages with respect to similar algorithms for the simulation of other dynamical systems, for instance the kicked rotator. There are no extra work space qubits, namely all the qubits are used to simulate the dynamics of the system. This implies that less than 40 qubits would be sufficient to make simulations inaccessible to present day supercomputers. We note that this figure has to be compared with the more than 1000 qubits required to the Shor's algorithm to outperform classical computations. We have also shown that in our model, dynamical localization should be observed already with 6 qubits.

Fidelity: we have studied the fidelity of the teleportation protocol in a noisy environment by means of the quantum trajectories method. Our studies demonstrate the ability of this approach to model quantum information protocols with a large number of qubits. This opens up many possibilities for future studies. Theoretical predictions for the behaviour of the fidelity and of other relevant quantities with respect to the system size and different kinds of environment can now be explored with the help of numerical simulations. It will be also possible to include the effects of realistic internal Hamiltonians. Since various quantum protocols can be easily modelled, the scope of the present approach can be extended to the study of their stability. Finally, quantum trajectories offer a very convenient framework to model experiments. In this context, we point out the ability of a single quantum trajectory to provide a good illustration of an individual experimental run. Therefore quantum trajectories promise to become a very valuable tool for quantum hardware design and to determine optimal regimes for the operability of quantum processors.

We have derived general semiclassical expressions for the fidelity decay, at strong perturbations, which reproduce, as particular limiting cases, previously obtained results. In particular we have discussed the relevance of fluctuations in the finite-time Lyapunov exponent and we have shown that fidelity decay depends on the strength of such fluctuations in the Lyapunov regime. We have also shown that under certain conditions the exponential rate of fidelity decay can be equal to twice the classical Lyapounov exponent.

A dynamical theory of fidelity decay has been reviewed and further developed, based on correlation function analysis of the unperturbed motion. The theory describes quantitatively the fidelity decay of Hamiltonian quantum dynamics perturbed with arbitrary static or noisy perturbation and works with arbitrarily accuracy in the so-called linear response regime. Namely, the theory is accurate in the regime where fidelity is close to 1 which is of main interest for quantum computation, whereas some regimes of asymptotic decay of fidelity can also be derived using additional assumptions. Specific cases of dynamical systems with chaotic or regular classical limits were studied in detail, and different universal regimes were described, depending on the strength of perturbation, effective value of Planck constant, nature of unperturbed classical dynamics and the type of the initial state. The theory predicts enhancement of fidelity with stronger correlation decay and has been applied to analyze and optimize fidelity of quantum algorithms. A particular situation of fidelity decay within dynamical systems’s framework has been described, which displays anomalously slow fidelity decay, and which has been termed ’quantum freeze of fidelity’. A particular class of the so-called residual perturbations has been identified, which reduce quantum fidelity qualitatively less than generic perturbations, and in certain regimes produce a high value of fidelity even saturated in time; the so-called plateau of fidelity. It has been discussed how such situations can naturally occur in physics, provided the unperturbed system and the perturbation posses certain geometric, dynamic, or anti-unitary symmetries (of opposite parities). We have discussed in detail semi-classical theory of fidelity freeze for the extreme cases where the classical dynamics is completely chaotic and regular. Possible application of fidelity freeze to quantum computation and the close relation to the theory of dynamical decoupling has been discussed.

A theory of classical fidelity has been developed for chaotic few and many body classical hamiltonian dynamics. These results may serve as a useful reference for quantum fidelity within the so-called Ehrenfest time-scale, but may also be of interest of its own in the classical statistical mechanics. The central idea of our theoretical approach is the use of classical interaction picture for the classical echo dynamics. In the few body case we have derived a cascade of exponential decays, on sufficiently short time-scale, whose rates are given by the spectrum of classical Lyapunov exponents, whereas in the many body case we find a super-exponential (doubly exponential) fidelity decay when approaching the thermodynamic limit.
A random matrix theory of fidelity decay has been developed and a minimal input random matrix model of fidelity decay has been proposed and solved within the linear response approximation. A simple result which predicts a universal form of the crossover between the so-called Fermi golden rule and Gaussian (perturbative) decays has been already successfully applied to various situation in different frameworks: (i) chaotic dynamical systems, (ii) theoretical models of quantum computation, and (iii) even in some experiments involving classical wave dynamics, like acoustics, elasticity or microwaves.

Entanglement: Entanglement is arguably the most peculiar feature of quantum systems, with no analog in classical mechanics. Furthermore, it is an important physical resource, which is at the basis of many quantum information protocols, including quantum cryptography and teleportation . For any quantum algorithm operating on pure states, the presence of multipartite (many-qubit) entanglement is necessary to achieve an exponential speedup over classical computation . Therefore the ability to control entangled states is one of the basic requirements for constructing quantum computers. In our work we have proposed a suitable method, the entanglement echo, to study the stability of entanglement under perturbations. We have shown that noise destroys the entanglement of a pair of qubits and produces entanglement between these two qubits and the other qubits of the quantum computer. We point out that, since the entanglement can be measured experimentally in an efficient way, entanglement echo experiments analogous to the numerical simulations discussed in this paper could be implemented in quantum processors with a small number of qubits (4-10) and a few hundreds of gates. These experiments are close to present capabilities and would bring new insights in our understanding of the limits to quantum computation due to decoherence and imperfections.
A dynamical theory has been developed in order to describe quantitatively the production of entanglement and decay of purity in weakly coupled composite systems. The theory relates the purity of reduced density matrix, as well as fidelity and the so-called reduced fidelity, of a general bi-partite composite system to the dynamical correlation functions of the coupling (perturbation) evolved by the uncoupled dynamics. Theoretical results were demonstrated by numerical experiments in coupled kicked tops, whose underlying classical mechanics ranged from integrable (regular) to strongly chaotic.

Listing of all publications

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C. Participating scientific personel

Prof. Carlo Beenakker
Prof. Tomaz Prozen
Dr. Giuliano Benenti
Dr. Philippe Jacquod
Dr. Joris van Velsen
Dr. M. Kindermann
Dr. Carlos Monasteiro
Dr. Gregor Veble
Dr. Marko Znidaric